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A Relationship between Turing Machines and Finite Automata on Four-Dimensional Input Tapes

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Abstract

We think that recently, due to the advances in computer animation, motion image processing, virtual reality systems, and so forth, it is useful for analyzing computation of multi-dimensional information processing to explicate the properties of four-dimensional automata. From this point of view, we first proposed four-dimensional automata in 2002, and investigated several accepting powers of them. In this paper, we coutinue the study, and mainly concentrate on investigating the relationship between the accepting powers of four-dimensional finite automata and seven-way four-dimensional tape-bounded Turing Machines.

Key Words **:** computation, finite automaton, four-dimensional, space complexity, Turing machine.

1 Introduction and Preliminaries

We think that recently, due to the advances in computer animation, virtual reality systems and so forth, it is useful for analyzing computation of multi-dimensional information processing to explicate the properties of four-dimensional automata . From this point of view, we first proposed four-dimensional automata in 2002 [3], and investiged several accepting powers of them [2,4,5]. In this paper, we continue the study, and show some results for open problems of four-dimensional finite automata.

Let Σ be a finite set of symbols. A *four-dimensional tape* over Σ is a four-dimensional array of elements of Σ. The set of all four-dimensional tapes over Σ is denoted by $\Sigma^{(4)}$. Given a tape $x \in \Sigma^{(4)}$, for each integer $j(1 \leq j \leq 4)$, we let $l_j(x)$ be the length of x along the jth axis. The set of all $x \in \Sigma^{(4)}$ with $l_1(x)=n_1$, $l_2(x)=n_2$, $l_3(x)=n_3$, and $l_4(x)=n_4$, is denoted by $\Sigma^{(n_1,n_2,n_3,n_4)}$. When $1 \leq i_j \leq l_j(x)$ for each $j(1 \leq j \leq 4)$, let $x(i_1,i_2,i_3,i_4)$ denote the symbol in x with coordinates (i_1,i_2,i_3,i_4). Furthermore, we define $x[(i_1,i_2,i_3,i_4),(i'_1,i'_2,i'_3,i'_4)]$, when $1 \leq i_j \leq i'_j \leq l_j(x)$ for each integer $j(1 \leq j \leq 4)$, as the four-dimensional input tape y satisfying the following conditions : (i) for each $j(1 \leq j \leq 4)$, $l_j(y) = i'_j - i_j + 1$; (ii) for each r_1, r_2, r_3, r_4, $(1 \leq r_1 \leq l_1(y)$, $1 \leq r_2 \leq l_2(y)$, $1 \leq r_3 \leq l_3(y)$, $1 \leq r_4 \leq l_4(y))$, $y(r_1,r_2,r_3,r_4) = x(r_1+i_1-1, r_2+i_2-1, r_3+i_3-1, r_4+i_4-1)$. We concentrate on the input tape x with $l_1(x) = l_2(x) = l_3(x) = l_4(x)$, throughout this paper, in order to increase the theoretical interest.

A *four-dimensional deterministic* (*nondeterministic*) *Turing machine* 4-*DTM* (or 4-*NTM*) M, which can be considered as a natural extension of the three-dimensional deterministic (or nondeterministic) Turing machine to four dimensions, consists of a read-only four-dimensional input tape, a finite control and one semi-infinite storage tape. A *step* of M consists of reading one symbol from each tape, writing a symbol on the storage tape, moving the input head in specified direction $d \in$ { east, west, south, north, up ,down, direction of the fourth axis, opposite direction of the fourth axis, no move }, moving the storage head in specified direction $d' \in$ { left, right, no move }, and entering a new state, in accordance with the next-move relation. An *SV*4-*DTM* (or *SV*4-*NTM*) is a 4-*DTM* (or 4-*NTM*) whose input head can move east, west, south, north, up, down, and in the direction of the fourth axis, but not in the opposite direction of the fourth axis (see Fig.1).

Let $L(n) : \mathbf{N} \rightarrow \mathbf{R}$ be a function of a variable n, where $\mathbf{N}$ is the set of all positive integers and $\mathbf{R}$ is the set of all nonnegative real numbers. A 4-*DTM* M is said to be $L(n)$ *space-bounded* if for no input tape $x \in \Sigma^{(4)}$ with $l_1(x) = l_2(x) = l_3(x) = l_4(x) = n$ does M scan more than $L(n)$ cells on the storage tape. We denote an $L(n)$ *space-bouned* 4-*DTM* (4-*NTM*) by 4-*DTM*($L(n)$) (4-*NTM*($L(n)$)). A 4-*DTM*(0) (4-*NTM*(0)) is called a *four-dimensional deterministic finite automaton* (*four-dimensional nondeterministic finite automaton*), and is denoted by 4-*DFA* (4-*NFA*). Let $\mathcal{L}$[4-*DTM*] $= \{T \mid T = T(M)$ for some 4-*DTM* M $\}$. $\mathcal{L}$[4-*NTM*], etc.

2 Main Results

In this section, we investigate the relationship between the accepting powers of 4-*XFA*'s and *SV*4-*YTM*'s for each $X, Y \in \{D, N\}$.

It is easy to see that the following lemma holds. So the proof is omitted here.

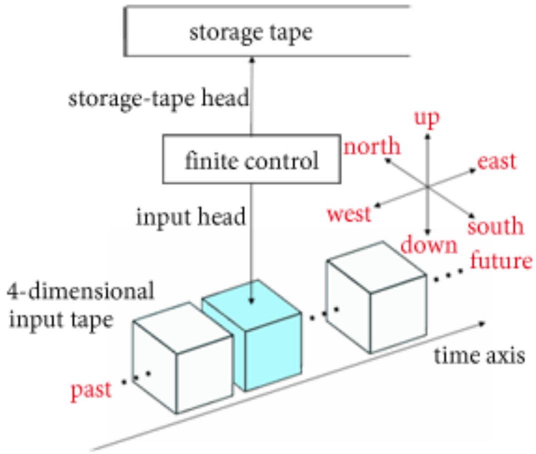

Fig. 1: Four-dimensional Turing machine.

Lemma 2.1. *Let* $T_1 = \{x \in \{0,1\}^{(4)}|^{\exists}\ n \geq 1\ [l_1(x) = l_2(x) = l_3(x) = l_4(x) = 2n{+}1\ \&\ x(n+1, n+1, n+1, n+1) = 1]\}$. *Then*

(1) $T_1 \in \mathcal{L}[SV4\text{-}DTM(\log n)]$, *and*

(2) $T_1 \in \mathcal{L}[SV4\text{-}NTM(0)]$.

Lemma 2.2. *Let* $T_2 = \{x \in \{0,1\}^{(4)}|^{\exists}\ n \geq 1$ $[l_1(x) = l_2(x) = l_3(x) = l_4(x) = 2n\ \&\ x[(1,1,1,1), (2n,2n,2n,n)]{=}x[(1,1,1,n+1),(2n,2n,2n,2n)]$*(that is, the top and bottom halves of x are identical)*$]\}$, *and let* $L(n){:}\mathbf{N}{\rightarrow}\mathbf{R}$ *be a function such that* $lim_{n\rightarrow\infty}[L(n)/n^4] = 0$. *Then*

(1) $T_2 \in \mathcal{L}[4\text{-}DTM(\log n)]$, *and*

(2) $T_2 \notin \mathcal{L}[SV4\text{-}NTM(L(n))]$.

Proof: It is easy to prove Part(1), and so the proof is left to the reader. We then prove Part(2).

Suppose that there exists some $SV4\text{-}NTM(L(n))$ M accepting T_2, and that q is the number of states of its finite control and t is the number of storage symbols. For each $n{\geq}1$, let

$V(n) = \{x \in T_2|\ l_1(x) = l_2(x) = l_3(x) = l_4(x) = 2n\}$.

Clearly, each tape in $V(n)$ is accepting by M. Furthermore, for each x in $V(n)$, let

conf$(x)\triangleq$the set of configurations of M just after the point, in the accepting computations on x, where the input head left the top half of x.

Then the following proposition must hold.

Proposition 2.1. *For any two different tapes x,y in* $V(n)$,

conf(x) $\cap$ conf(y) $= \phi$ *(empty set)*.

[For otherwise, suppose that conf$(x) \cap$ conf$(y) {\neq} \phi$ and $\sigma \in$ conf$(x) \cap$ conf(y). It is obvious that if, starting with this configuration σ, the input head proceeds to read the bottom half of x, then M could enter an accepting state. Therefore, by assumption, it follows that the tape z satisfying the following three conditions must be also accepted by M:

(i) $l_1(z) = l_2(z) = l_3(z) = l_4(z) = 2n$;

(ii) $z[(1,1,1,1),(2n,2n,2n,n)] = y[(1,1,1,1), (2n,2n,2n,n)]$;

(iii) $z[(1,1,1,n+1),(2n,2n,2n,2n)] = x[(1,1,1,n+1),(2n,2n,2n,2n)]$.

This contradicts the fact that z is not in T_2, and thus the proposition holds. □]

Clearly, $|V(n)| = 2^{2n\cdot 2n\cdot 2n\cdot n} = 2^{8n^4}$. On the other hand, let $c(n)$ be the number of possible configurations of M just after the input head left the top halves of tapes in $V(n)$. Then we get the following inequality:

$c(n){\leq}q{\cdot}(2n+2)^3{\cdot}L(2n){\cdot}\ t^{L(2n)}$.

Since $lim_{n\rightarrow\infty}[L(2n)/16n^4] = 0$ (by the assumption of the lemma), $|V(n)|{>}c(n)$ for large n. Therefore, it follows that for large n there must be two different tapes x and y in $V(n)$ such that conf$(x) \cap$ conf$(y){\neq}\phi$. This contradicts Proposition 2.1, and thus Part(2) of the lemma holds. □

Lemma 2.3. *Let* $T_3 = \{x \in \{0,1\}^{(4)}|^{\exists}\ n < 1$ $[l_1(x) = l_2(x) = l_3(x) = l_4(x) = 2n\ \&\ x[(1,1,1,1), (2n,n,2n,2n)] = x[(1,n+1,1,1),(2n,2n,2n,2n)]]\}$. *Then*

(1) $T_3 \in\mathcal{L}[SV4\text{-}DTM(\log n)]$, *and*

(2) $T_3 \notin\mathcal{L}[4\text{-}NFA]$.

Proof: The proof of (1) is omitted here, since it is obvious. We now prove (2). Suppose that T_3 is in $\mathcal{L}[4\text{-}NFA]$. By using the some idea as in the proof of Lemma 4.3 in [6], it follows that T_2 is also in $\mathcal{L}[4\text{-}NFA]$. From this fact and the same technique as in the proof of Lemma 3.5 in [6], it follows that T_2 is in $\mathcal{L}[SV4\text{-}NTM(n^3)]$. This contradicts Lemma 2.2(2), and thus Part(2) of the lemma holds. □

It is obvious from definitions that the following lemma holds.

Lemma 2.4.

(1) $\mathcal{L}[SV4\text{-}DTM(0)] \subseteq \mathcal{L}[4\text{-}DFA]$.

(2) $\mathcal{L}[SV4\text{-}NTM(0)] \subseteq \mathcal{L}[4\text{-}NFA]$.

From the above lemmas, we can prove the following theorem concerning the relationship between the accepting powers of 4-XFA's ($X \in \{D, N\}$) and $SV4$-DTM's.

Theorem 2.1.

(1) $\mathcal{L}[4\text{-}DFA] \subsetneq \mathcal{L}[SV4\text{-}DTM(n^3 \log n)]$.

(2) $\mathcal{L}[4\text{-}NFA] \subsetneq \mathcal{L}[SV4\text{-}DTM(n^4)]$.

(3) *For any function* $L(n)$:$\mathbf{N} \rightarrow \mathbf{R}$ *such that* $L(n) \geq \log x$ $(n \geq 1)$,

(i) *if* $lim_{n\rightarrow\infty}[L(n)/n^3 \log n]$=0, *then* $\mathcal{L}[3\text{-}DFA]$=0, *then* $\mathcal{L}[3\text{-}DFA]$ *is incomparable with* $\mathcal{L}[SV4\text{-}DTM(L(n))]$, *and*

(ii) $lim_{n\rightarrow\infty}[L(n)/n^4]$=0, *then* $\mathcal{L}[4\text{-}NFA]$ *is incomparable with* $\mathcal{L}[SV4\text{-}DTM(L(n))]$.

(4) $\mathcal{L}[SV4\text{-}DTM(0)] \subsetneq \mathcal{L}[4\text{-}DFA]$.

Proof: By using the same technique as in the proof of Lemma 3.1 in [6], $\mathcal{L}[4\text{-}DFA] \subseteq \mathcal{L}[SV4\text{-}DTM(n^3 \log n)]$. From this fact, Lemma 2.1(1), and the same technique as in the proof of Lemma 2.1(2) in [6], it follows that $\mathcal{L}[4\text{-}DFA] \subsetneq \mathcal{L}[SV4\text{-}DTM(n^3 \log n)]$ and thus Part(1) of the theorem holds. Similarly, Part(2) can be proved by using Lemma 2.3 and the same technique as in the proof of Lemma 3.3 in [6], and Part(4) can be proved by using Lemma 2.4(1) and the same technique as in the proof of Lemma 3.2 in [6].

We then prove Part(3). From Lemma 2.1(1), and the same technique as in the proof of Lemmas 2.1(2) and 3.2 in [6], it follow the Case(i) of Part(3) holds. Case(ii) in also proved by using Lemma 2.3 and the same technique as in the proof of Lemma 3.4 in [6]. □

Remark 2.1. By using the same idea as in the proof of Lemma 3.2 in [6], we can show that there is no result which is stronger than Theorem 2.1(1), and by using the same idea as in the proof of Lemma 3.4 in [6], we can show that there is no result which is stronger than Theorem 2.1(2).

We next give the following theorem concerning the relationship between the accepting powers of 4-XFA's($X \in \{D, N\}$) and $SV4$-NTM's.

Theorem 2.2.

(1) $\mathcal{L}[4\text{-}NFA] \subsetneq \mathcal{L}[SV4\text{-}NTM(n^3)]$.

(2) *Let* $L(n)$:$\mathbf{N} \rightarrow \mathbf{R}$ *be a function such that* $lim_{n\rightarrow\infty}[L(n)/n^3]$=0. *Then* $\mathcal{L}[4\text{-}DFA]$ *is incomparable with* $\mathcal{L}[SV4\text{-}NTM(L(n))]$.

(3) *Let* $L(n)$:$\mathbf{N} \rightarrow \mathbf{R}$ *be a function such that* $L(n) \geq \log n$ $(n \geq 1)$ *and* $lim_{n\rightarrow\infty}[L(n)/n^3]$=0. *Then* $\mathcal{L}[4\text{-}NFA]$ *is incomparable with* $\mathcal{L}[SV4\text{-}NTM(L(n))]$.

(4) $\mathcal{L}[SV4\text{-}NTM(0)] \subsetneq \mathcal{L}[4\text{-}NFA]$.

Proof: By using the same technique as in the proof of Lemma 3.5 in [6], $\mathcal{L}[4\text{-}NFA] \subseteq \mathcal{L}[SV4\text{-}NTM(n^3)]$. From this fact and Lemma 2.3, it follows that $\mathcal{L}[4\text{-}NFA] \subsetneq \mathcal{L}[SV4\text{-}NTM(n^3)]$, and thus Part(1) of the theorem holds. Similarly, Part(4) can be proved by Lemma 2.4(2) and the same technique as in the proof of Lemma 3.6 in [6]. We can then proof Part(2) by using Lemma 2.1(2) and the same technique as in the proof of Lemmas 2.1(2) and 3.6 in [6]; and Part(3) is also proved by using Lemma 2.3 and the same technique as in the proof of Lemma 3.6 in [6]. □

Remark 2.2. By using the same idea as in the proof of Lemma 3.6 in [6], we can show that there is no result which is stronger than Theorem 2.2(1).

We complete this section by giving the following theorem concerning the relationship between the accepting powers of 4-NFA's and $SV4$-DTM's.

Theorem 2.3. $\mathcal{L}[4\text{-}NFA] \subsetneq \mathcal{L}[SV4\text{-}DTM((\log n)^2)]$.

Proof: It is obvious from Lemma 2.3 and the same technique as in the proof of theorem 3.5 in [6] that the theorem holds. □

3 Conclusion

In this paper, we mainly concentrated on investigating the relationship between the accepting powers of four-dimensional finite automata and seven-way four-dimensional tape-bounded Turing machines.

It will be interesting to investigate the relationship between the accepting powers of alternating finite automata and Turing machines on four-dimensional input tapes (see [1] for the concept of *alternation*).

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